

Brillouin scattering and the CMB

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Abstract

Brillouin scattering of photons off the density fluctuations in a fluid is potentially important for cosmology. We derive the Brillouin spectral distortion of blackbody radiation, and discuss the possible implications for the cosmic microwave background. The thermal Sunyaev-Zeldovich effect is slightly modified by Brillouin distortion, but only at very long wavelengths.

1 Introduction

Dynamical light scattering theory [1] shows that photons interact with statistical fluctuations in the fluid density and temperature. The origin of these fluctuations can be traced back to the fact that classical particles possess random trajectories that allow a description in terms of a dynamic structure factor. Early theoretical work by Brillouin [2] predicted a doublet in the frequency distribution of monochromatic light scattered by sound waves in a fluid. Experimental work [3] soon confirmed the Brillouin doublet about the central or Rayleigh line. Landau and Placzek [4] gave a theoretical explanation of the Brillouin spectrum, by means of a thermodynamical approach. Brillouin scattering has been mainly investigated in the context of laboratory physics. We are not aware of applications to cosmology, although Brillouin scattering has been considered in planetary atmospheres [5] and pulsar eclipses [6].

In this paper, we derive the Brillouin spectral distortion of blackbody radiation interacting with a hydrodynamical fluid. We discuss the possible applications of this result to the cosmic microwave background (CMB) radiation. We also investigate the interaction of the CMB with hot intra-cluster gas. The thermal Sunyaev-Zeldovich (SZ) effect is slightly modified by Brillouin scattering, but only at very long wavelengths: the CMB is “protected” from thermal Brillouin distortion by the low density and high temperature of the intra-cluster gas, which means that it does not behave like a hydrodynamical fluid.

2 The Brillouin spectrum

The standard system describing fluctuations of the thermodynamical variables in a hydrodynamical fluid of particles of mass m can be written as [1]:

$$\frac{\partial}{\partial t} \delta n + n_o \delta \Theta = 0, \quad (1)$$

$$\frac{\partial}{\partial t} \delta \Theta + \frac{c_s^2}{n_o} \nabla^2 \delta n + \alpha c_s^2 \nabla^2 \delta T = 0, \quad (2)$$

$$\frac{\partial}{\partial t} \delta T + \left(\frac{1-\gamma}{\alpha n_o} \right) \frac{\partial}{\partial t} \delta n = 0, \quad (3)$$

where δn is the fluctuation of the particle density relative to its average value n_o , $\Theta = \nabla \cdot \vec{v}$ is the fluid expansion rate, c_s^2 is the sound speed, $\alpha = -n^{-1}(\partial n / \partial T)_p$ is the thermal expansion coefficient, δT is the temperature fluctuation, and $\gamma = C_p / C_V$ is the heat capacity ratio.

Following Landau and Placzek [4], a spatial Fourier transform is taken of Eqs. (1)–(3) and then a Laplace transform is performed with respect to time:

$$\begin{bmatrix} s & n_o & 0 \\ -c_s^2 k^2 / (\gamma n_o) & s & -\alpha c_s^2 k^2 / \gamma \\ (1-\gamma)s / (\alpha n_o) & 0 & s \end{bmatrix} \begin{bmatrix} \delta \tilde{n}(k, s) \\ \delta \tilde{\Theta}(k, s) \\ \delta \tilde{T}(k, s) \end{bmatrix} = \begin{bmatrix} \delta \tilde{n}(k, 0) \\ \delta \tilde{\Theta}(k, 0) \\ \delta \tilde{T}(k, 0) + (1-\gamma)\delta \tilde{n}(k, 0) / (\alpha n_o) \end{bmatrix}. \quad (4)$$

The solution for the density fluctuations in Fourier space is

$$\delta \tilde{n}(k, t) = \delta \tilde{n}(k, 0) \left[\left(1 - \frac{1}{\gamma} \right) + \frac{1}{\gamma} \cos(c_s k t) \right]. \quad (5)$$

The Brillouin specific intensity of the scattered light due to its interaction with acoustic modes of a fluid is given by

$$I_\omega \propto \int_{-\infty}^{\infty} \langle \delta \tilde{n}(k, 0)^* \delta \tilde{n}(k, t) \rangle e^{-i\omega t} dt, \quad (6)$$

which leads to

$$I_\nu \propto \left(1 - \frac{1}{\gamma} \right) \delta(\nu) + \frac{1}{2\gamma} \delta\left(\nu + \frac{c_s}{c}\nu\right) + \frac{1}{2\gamma} \delta\left(\nu - \frac{c_s}{c}\nu\right). \quad (7)$$

The Brillouin spectrum is a sum of Dirac delta functions. In the presence of dissipative effects, the delta peaks acquire finite height and width, but we are neglecting non-equilibrium effects, which would typically (but not always) produce only a small correction to our equilibrium results. The first term in Eq. (7) represents the Rayleigh peak. The next two terms are the Brillouin doublet. This doublet reflects a shift in frequency governed by the speed of sound of the fluid, directly related to the Doppler effect, as shown by Eq. (5):

$$\Delta\nu = \frac{c_s}{c}\nu. \quad (8)$$

The adiabatic speed of sound for a monatomic gas is given by

$$c_s = \frac{k_B T}{m} \gamma. \quad (9)$$

The integrated intensities under each peak are simply related by the Landau-Placzek ratio. If we denote by I_s the area under the singlet peak, and I_d the area under one of the doublet peaks, then

$$I = \int_{-\infty}^{\infty} I_\omega d\omega = I_s + 2I_d, \quad (10)$$

where

$$\frac{I_s}{I_d} = \gamma - 1. \quad (11)$$

Thus, for a monatomic gas with $\gamma = \frac{5}{3}$, it follows that $\frac{1}{4}$ of the total intensity of light scattered from a fixed frequency ν will remain unshifted, while $\frac{3}{4}$ of the intensity will be shifted in equal parts to the frequencies $\nu \pm \Delta\nu$, given by Eq. (8). Also, if the density of the fluid increases, $\gamma \rightarrow 1$ and the Rayleigh peak disappears, leaving all the scattered radiation divided by equal parts in the Brillouin doublet.

In the hydrodynamical limit, density fluctuations propagate as sound waves and cause the doublet. In this limit, sound waves of all wavenumbers are possible, so that all photon frequencies are affected by Brillouin scattering. We will discuss later how this is changed in the non-hydrodynamical regime.

The undistorted blackbody spectrum is

$$I = 2 \frac{(k_B T_r)^3}{(hc)^2} F(x), \quad (12)$$

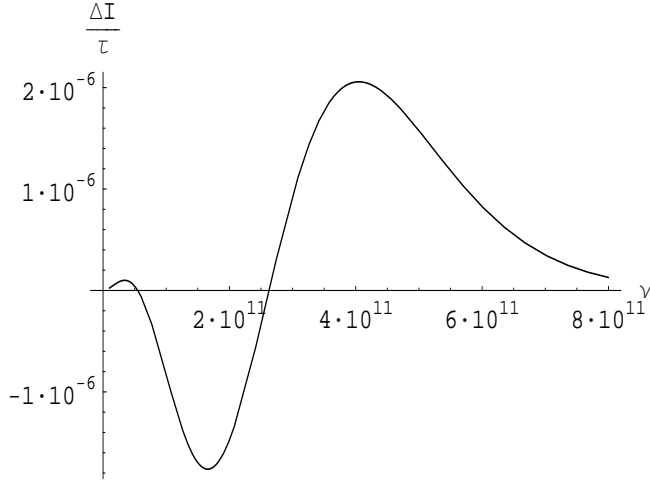


Figure 1: CMB distortion for the case of Brillouin scattering off hydrodynamical acoustic modes, as given by Eq. (16). The fluid temperature is $T = 10^4$ K. The frequency is given in Hz and $\Delta I_{\text{br}}/\tau_{\text{br}}$ is given in units of $2(kT_r)^3/(hc)^2$.

where

$$x = \frac{h\nu}{k_B T_r}, \quad F(x) = \frac{x^3}{e^x - 1}, \quad (13)$$

and T_r is the radiation temperature. The effect of Brillouin scattering on the intensity spectrum can be calculated from Eqs. (7)–(11), for the case $\gamma \rightarrow 1$, by means of the convolution integral [7]:

$$I_{\text{br}}(\nu) = [1 - \tau_{\text{br}}]I(\nu) + \frac{\tau_{\text{br}}}{2} \int_{-\infty}^{\infty} I(\bar{\nu}) \left\{ \delta\left(\bar{\nu} - \nu \left[1 + \frac{c_s}{c}\right]\right) + \delta\left(\bar{\nu} - \nu \left[1 - \frac{c_s}{c}\right]\right) \right\} d\bar{\nu}. \quad (14)$$

Here τ_{br} is the fraction of photons that are Brillouin scattered off the acoustic modes of the fluid and $I_{\text{br}} = I + \Delta I_{\text{br}}$ is the perturbed spectrum. For a fairly dense system, $\tau_{\text{br}} \sim 1$.

The Brillouin peaks are Dirac delta functions, preserving the intensity fractions of the scattered radiation. If we now perform the substitution

$$\alpha = \bar{\nu} - \nu \left(1 \pm \frac{c_s}{c}\right), \quad (15)$$

we may expand the integrand in a Taylor series. Straightforward calculation leads to the following expression for CMB scattering off the acoustic modes:

$$\Delta I_{\text{br}} = \frac{\tau_{\text{br}}}{2} \left(\frac{c_s}{c}\right)^2 \nu^2 \frac{\partial^2 I}{\partial \nu^2}. \quad (16)$$

This expression is plotted in Fig. 1 for $c_s = 5.7 \times 10^7$ cm/s, which roughly corresponds to a fluid temperature of 4000 K. The curve resembles the ordinary thermal SZ effect, with the "crossover frequency" in this case being $\nu_{\text{a,c}} = 262.612$ GHz.

3 Corrections to the SZ effect?

Consider now the case of CMB photons crossing the intra-cluster gas of a galaxy cluster. The Brillouin distortion is apparently given by Eq. (16), which may be compared with the (non-relativistic, thermal) SZ distortion [8]

$$\Delta I_{\text{sz}} = yI \frac{e^x F(x)}{2x^2} \left[x \coth \frac{x}{2} - 4 \right]. \quad (17)$$

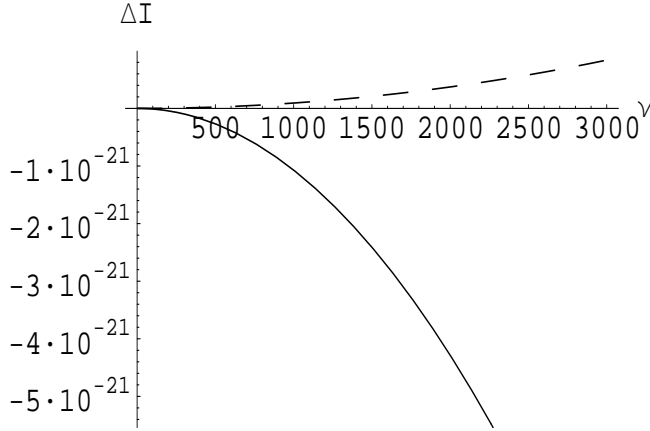


Figure 2: The solid line represents the ordinary SZ effect in the given frequency range. The dashed line corresponds to Brillouin scattering. The cluster parameters are $T = 10^7$ K, and optical depth $\tau = 10^{-3}$. The units are as in Fig. 1.

However, the intra-cluster gas is hot ($T \sim 10^8$ K) and very dilute ($n \sim 10^3 \text{ m}^{-3}$). This means that it is far from the hydrodynamic (collision-dominated) regime, and much closer to the collisionless regime. Therefore the hydrodynamical analysis above does not cover the intra-cluster gas. Pressure fluctuations do not disappear, but they no longer propagate as sound waves at all frequencies. While there may be sound-like waves, there will be a maximum frequency, above which there is no propagation [9].

An interesting question is how the transition from propagating to non-propagating frequencies is determined, but answering this question requires a detailed kinetic-theory analysis, which will be tackled elsewhere. For present purposes, we can assume that there is an instantaneous cut-off at about the plasma frequency. For typical clusters, this gives an upper limit

$$k \leq k_{\text{max}} \sim k_{\text{plasma}} \sim 10^{-4} \text{ m}^{-1}. \quad (18)$$

This means that standard Brillouin scattering of the CMB by clusters is cosmologically insignificant, since it affects only very low frequencies.

Nevertheless it is interesting to compute the distortion up to the cut-off frequency, which is given by Eq. (7), subject to Eq. (18), in the case $\gamma = \frac{5}{3}$. The corresponding result now reads:

$$\Delta I = \frac{1}{4\gamma} \left(\frac{c_s}{c} \right)^2 \nu^2 \frac{\partial^2 I}{\partial \nu^2}. \quad (19)$$

The Brillouin distortion of the CMB by a typical cluster over the allowable range of frequencies is shown in Fig. 2. The SZ distortion in this range is also shown, for comparison. Although the Brillouin contribution is not negligible, it operates at frequencies which are of little interest.

4 Discussion

We derived the distortion of a blackbody spectrum that follows from the hydrodynamical Brillouin frequency shift, given in Eq. (16) and illustrated in Fig. 1. The question then is what are the cosmological implications of this result. We considered the Brillouin distortion induced by clusters, but since the intra-cluster gas is not hydrodynamical, the standard analysis does not apply beyond a very low frequency, so that Brillouin distortion is insignificant.

The ordinary Brillouin spectrum arises from light scattering off pressure fluctuations at constant entropy. When the fluid is collision-dominated, these fluctuations propagate as sound waves and generate a Brillouin doublet for each wavenumber. As density decreases (and the Knudsen number approaches 1), the concept of sound propagation eventually becomes meaningless. For clusters, there is a cut-off maximum wavenumber for propagating modes, of roughly the plasma frequency, i.e. $\sim 10^{-4} \text{ m}^{-1}$. Thus the CMB is “protected” from Brillouin distortion over all frequencies except the very low tail. The SZ effect is therefore in practice unaffected by ordinary Brillouin scattering.

Nevertheless, there may still be some Brillouin-type effects on the CMB from clusters. Pressure fluctuations still exist beyond the hydrodynamic regime and, although no doublet is present in the scattered spectrum, each central (Rayleigh) line will be broadened by an quantity proportional to the “width parameter” [10]

$$s(\nu) = \frac{2\pi\nu}{c} \left(\frac{k_B T}{m} \right)^{1/2}. \quad (20)$$

This means that some scattering of photons will occur due to pressure fluctuations even in the collisionless limit. The distortion induced by this broadening is of the form given by the convolution integral (compare [11])

$$I + \Delta I \propto \int_0^\infty \frac{I(\nu')}{s(\nu')} \exp - \left[\frac{\nu' - \nu}{s(\nu')} \right]^2 d\nu'. \quad (21)$$

This non-standard collisionless Brillouin distortion could lead to effects at significant frequencies, and this is currently under investigation.

A Boltzmann equation approach to the study of time correlation functions shows that sound-wave peaks are present for wavelengths comparable to the mean free path [12]. Furthermore, because of the high densities of hot photons in the intra-cluster gas, there may be propagating modes at higher frequencies; relativistic plasmas at high temperatures ($T > 10^8 \text{ K}$) may in some sense be treated as collision-dominated systems [13]. Other arguments that suggest collective behaviour of low density plasmas far beyond the plasma frequency have been developed in [14]: the plasma-dynamical regime describes motions of spatially smooth, weakly damped disturbances which may oscillate at high frequencies.

In view of these points, we believe that dynamical light scattering of CMB photons in clusters should not be simply dismissed, and further investigation is warranted. Furthermore, there remains the potentially more important question of whether ordinary Brillouin scattering has an effect during recombination.

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